Greene's Theorem

Robinson-Schensted Correspondence

perm $S_n \ni w \longmapsto (P, Q)$

SYTs of the same shape
$\lambda = (\lambda_1, \dots, \lambda_n)$
called (Schensted) shape of
permutation w

Thm For $r = 1, 2, \dots$

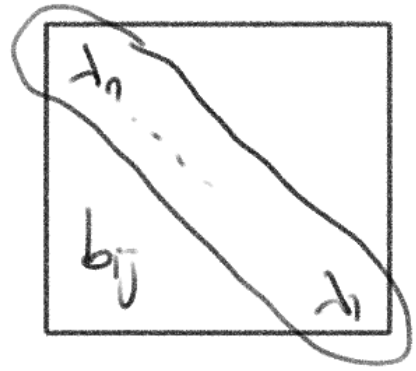
$\boxed{1}$ $\lambda_1 + \dots + \lambda_r$ is the maximal size of disjoint increasing subsequences of w.

$\boxed{2}$ $\lambda_1' + \dots + \lambda_n'$ is the maximal size of disjoint decreasing subsequences of w.

Generalization of RSK

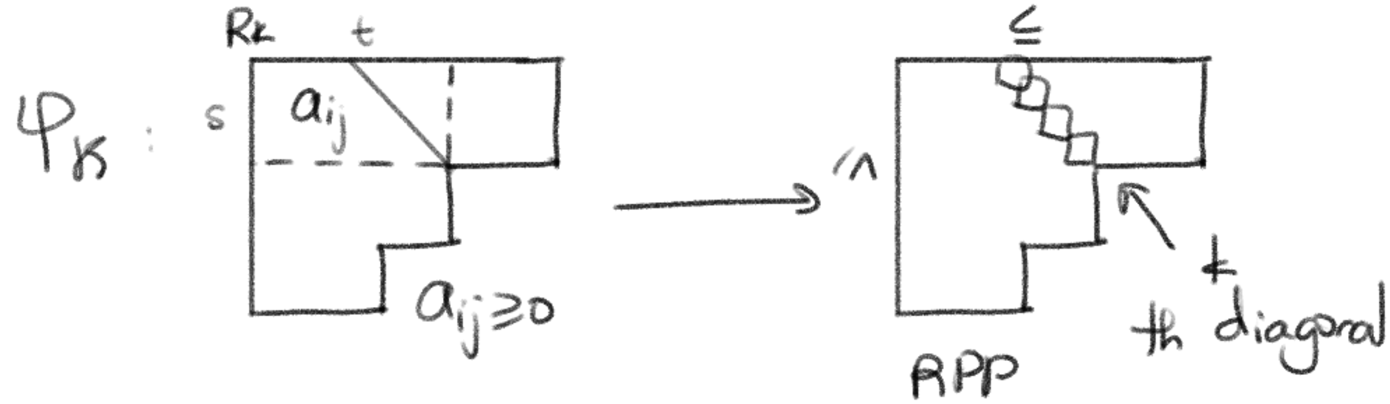
$\boxed{a_{ij} \geq 0} \xrightarrow{\text{RSK}} (P, Q)$ SSYT-s of some shape

$\Big\downarrow$ GT patterns



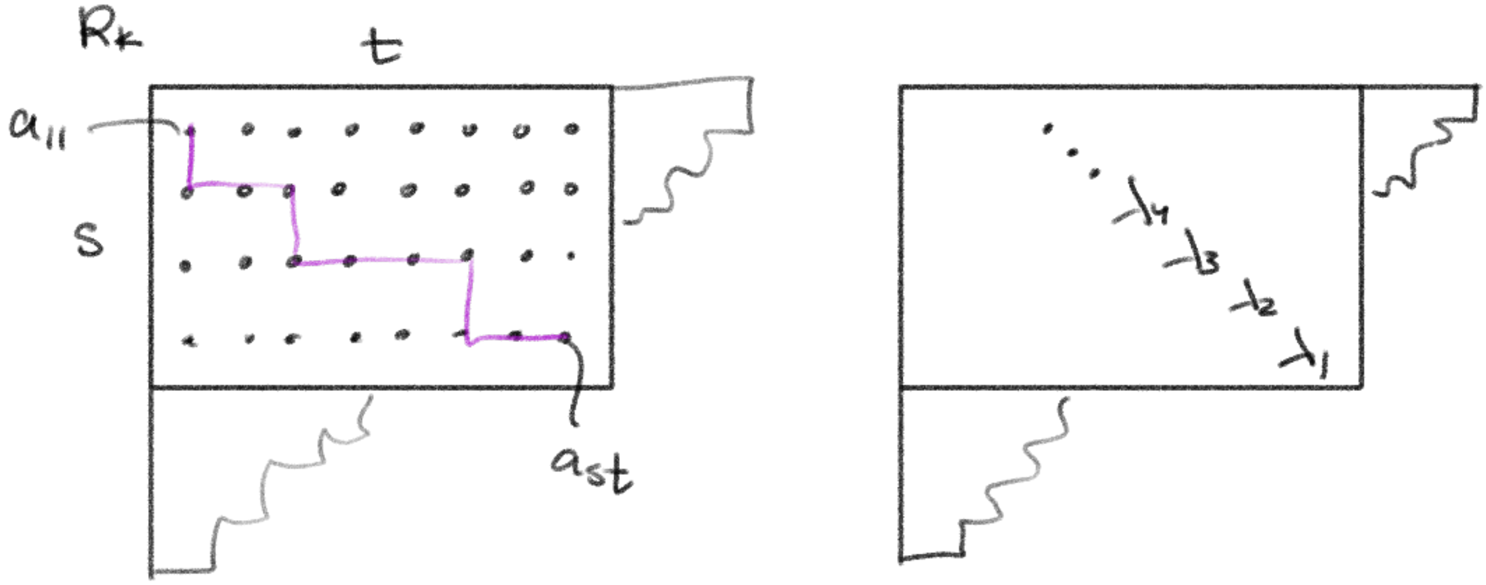
RPP-s

Even more generally



The rule only depends on $s \times t$ rectangle R_k

Generalized (Greene's Theorem)



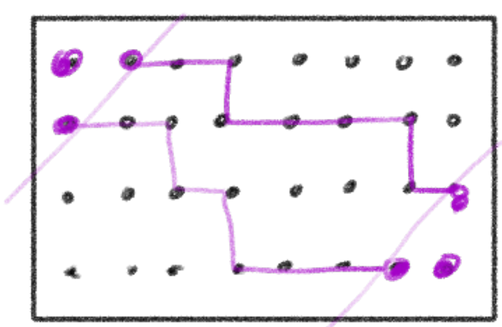
$$\lambda_1 = \max_{P} \left(\sum_{(i,j) \in P} a_{ij} \right)$$

$\hookrightarrow$ Lattice paths from a_{11} to a_{st}.

$$\lambda_1 + \lambda_2 = \max_{P_1, P_2} \left(a_{11} + a_{st} + \sum_{(i,j) \in P_1 \cup P_2} a_{ij} \right)$$

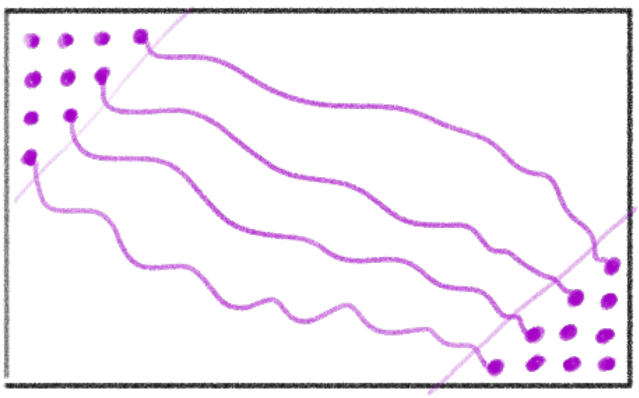
$\hookrightarrow$ two non-crossing lattice paths

$P_1 : a_{12} \rightsquigarrow a_{s-1,t}$ $\quad$ $P_2 : a_{21} \rightsquigarrow a_{s,t-1}$



In general, for r

$\lambda_1 + \cdots + \lambda_k \rightsquigarrow$



Tropical Calculus

subtraction-free Rational Calculus	Tropical Calculus
$A \cdot B$	$a + b$
A / B	$a - b$
$A + B$	$\max\{a, b\}$
const	0